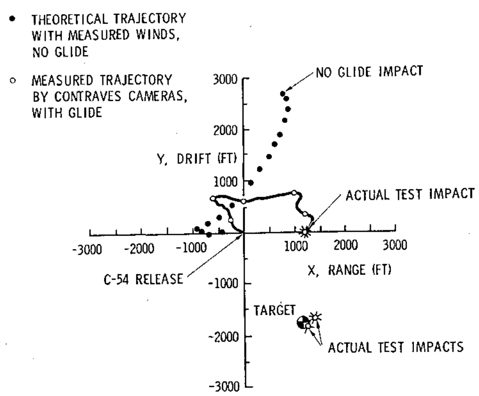

Fig. 3 Plan view of guided gliding drop tests.

control was incorporated. An analog computer program was devised‡ to study the effects of proportional control on stability and steering accuracy and on the effects of using a fixed camera and gyrostabilized camera as steering aids. The proportional control system was evaluated through another series of drop tests. Controllability was greatly improved.

To get a shorter "turn around" time and to visually observe the effects of rigging changes, a platform was built on a diesel truck trailer, and tests were conducted at speeds of 50–60 mph. Better methods of routing control lines and improving stability were developed during these tests at considerable savings.

Further drop tests were then made to verify the rigging changes and the use of a downward-pointed TV camera in the nose of the vehicle to help the operator steer toward a selected impact point. Both fixed and gyro-stabilized cameras were used. This improved operator accuracy and allowed remote operation desirable for certain applications. No radar or visual acquisition of the vehicle was necessary, allowing a simpler over-all system.

Results

A plan view of the flight path of a representative drop test is shown in Fig. 3. The no-glide impact shown was computed by using the measured winds and the actual vertical speed measured on the drop. Drop altitude from the C-54 aircraft was 20,000-ft MSL, and target altitude was approximately 5500-ft MSL. Nearly 3000 ft of wind drift was cancelled by using the controlled glide capability of the parachute.

Circular errors of 1800 ft, 240 ft and 111 ft have been obtained, as shown in Fig. 3, for a release altitude of 20,000-ft MSL. It is believed that accuracy of this system can be developed to 50-ft CEP, or better. Operationally, the controller with a television monitor would be located in an aircraft above the desired impact point.

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‡ This work is done by H. R. Vaughn, Supervisor of Aeroballistics Division, Aerothermodynamics Projects Department.

Calculation of Supersonic Compressor Losses

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Nomenclature

A_{I}/A_{II}	=	$S/S + \tau$ = dump area ratio
c	=	chord
f	=	correction factor
g	=	$(P_t)_2/(P_t)_1$
h	=	radial blade length
M	=	Mach number
P	=	pressure
S	=	blade spacing
γ	=	ratio of specific heats
ϵ	=	$f_n - f$
θ	=	blade camber angle
η	=	stage adiabatic efficiency
σ	=	c/S = solidity
τ	=	trailing edge thickness
ξ	=	cascade stagger angle, measured from axial direction

Subscripts

0	=	subsonic conditions
1	=	conditions upstream ahead of cascade inlet
2	=	condition's downstream of cascade
n	=	nominal conditions
s	=	conditions behind shock wave
t	=	total conditions

I. Introduction

THE theoretical prediction of supersonic flow properties in a curved channel or between adjacent blades of a cascade presents a formidable task because of the complex nature of the interaction of shock waves, the vortex sheets, and the boundary layer.[1,2] A more reliable means of obtaining the performance of these compressors is experimental testing, as described in Refs. 3–7. For design purposes the idea of a simple analytical or semiempirical means of predicting the performance is attractive. Several papers along these lines have appeared recently in the literature, among them is the work of Balzer[8] in which the boundary-layer blockage effect and change in shock position are accounted for. A semiempirical method for predicting the performance of high reaction supersonic compressor blade sections is given by Boxer.[9]

The present analysis is an attempt to supplement the previous analyses with still another semiempirical performance estimation that is believed to be simpler in application and more widely applicable to a large family of cascade geometries. The success of the method is due, in part, to the manner in which the experimental data was used to determine the initial Mach number influence on a key parameter in the efficiency expression. This method extends a successful formulation developed for subsonic compressors by Losey and Tabakoff[10] to the cases of supersonic compressors in which shock losses are present and accounted for. Other effects, such as dump losses and errors in the shock structure model used, are partially accounted for through the use of the experimental data.

II. Mathematical Flow Model

The purpose of this analysis is the development of a simple realistic means to compute the adiabatic efficiency of a super-

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sonic compressor. With this in mind, several simplifications and reasonable assumptions are made. It is assumed that the flow through the cascade is one dimensional in the axial direction, and the fluid properties are constant up to the shock position and equal to the value along the mean flow path. It is also assumed that the complicated boundary-layer pseudo shock wave interaction pattern between adjacent blades can be approximated by a normal shock standing at the leading edge of the pressure blade (see Fig. 1). It is further assumed that for the range of back pressures treated, the variation in shock position is unimportant. However, these results may be extended easily to account for a wider range in back pressure variation by introducing the pressure ratio across the cascade as an additional parameter in the analysis.

The adiabatic efficiency associated with a stage of a compressor is expressed in terms of the initial and final conditions as

$$\eta = \frac{\{1 + [(\gamma - 1)/2]M_1^2\}[(P_t)_2/(P_t)_1]^{(\gamma-1)/\gamma} - 1}{(\gamma - 1)/2M_1^2} \tag{1}$$

The total pressure ratio across the cascade is written as the product $[(P_t)_2/(P_t)_s][(P_t)_s/(P_t)_1]$. The ratio $(P_t)_s/(P_t)_1$ is assumed to vary according to the normal shock relation,

$$\frac{(P_t)_s}{(P_t)_1} = \left[\frac{(\gamma + 1)M_1^2}{(\gamma - 1)M_1^2 + 2}\right]^{\gamma/\gamma-1} \times \left[\frac{\gamma+1}{2\gamma M_1^2 - (\gamma - 1)}\right]^{1/\gamma-1} \tag{2}$$

The underlying idea in this analysis is concerned with the determination of the ratio $(P_t)_2/(P_t)_s$.

It is assumed that, to first order, the flow behind the normal shock is similar to the flow through a subsonic cascade. To account for the presence of the interacting shock wave and boundary layer in the cascade, a correction factor f is introduced into the expression for the efficiency of the subsonic cascade, i.e.,

$$\eta_0 = \frac{\{1 + [(\gamma - 1)/2]M_s^2\}[(P_t)_2/(P_t)_s]^{(\gamma-1)/\gamma} - 1}{f[(\gamma - 1)/2]M_s^2} \tag{3}$$

Table 1 Data for supersonic cascades

	CAMBER	SOLIDITY	THICKNESS RATIO	SYMBOL	REF.
M_1=1.0, 1.2, 1.3, 1.4 ξ = 55° τ = 0 IN. A_I/A_{II}=1.00	15° 15° 15° 10° 10° 10°	1.00 1.18 1.43 1.00 1.18 1.43	0.060 0.060 0.060 0.063 0.063 0.063	▲ △ ▽ • ○ ⊗	6 6 6 5 5 5
M_1=1.46 ξ =50° τ =0.416 IN. A_I/A_{II}=0.70	30°	3.17	0.136	■	6
M_1=1.50 ξ =45° τ =0.416 IN. A_I/A_{II}=0.70	30°	3.17	0.136	□	7
M_1=1.51 ξ =50° τ =0.416 IN. A_I/A_{II}=0.70	30°	3.17	0.136	■	6
M_1=1.58 ξ =50° τ =0.416 IN. A_I/A_{II}=0.70	30°	3.17	0.136	■	6
M_1=1.70 ξ =60° τ =0.141, 0.189, 0.206 IN. A_I/A_{II}=0.799, 0.746, 0.740	30°	3.00	0.084 0.1122 0.123	+ ◓ ◒	3 3 3
M_1=1.71 ξ =45° τ =0.416 IN. A_I/A_{II}=0.70	30°	3.17	0.136	□	7

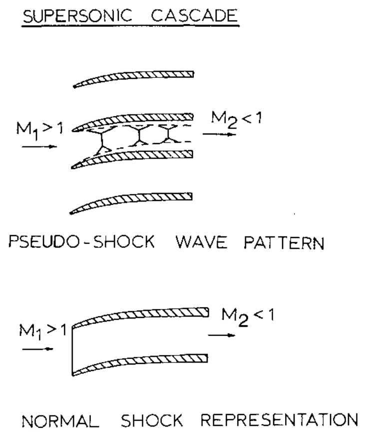

Fig. 1 Normal shock representation of the pseudo-shock wave pattern.

This expression provides a means of computing the ratio $(P_t)_2/(P_t)_s$ since η_0 can be computed from (see Ref. 10)

$$\left(\frac{1 - \eta_0}{0.14\eta_0}\right)^2 = \frac{\left[\dfrac{(\sigma)^{1/2}(c/h + \sigma)}{\cos\xi \cos^2\theta}\right]^{0.75}}{\left[\log_{10}\left(\dfrac{R}{\sigma^{0.25}\cos^2\theta}\right)\right]^{1.25}\left(1 + \dfrac{\gamma - 1}{2} M_s^2\right)^{0.226}} \tag{4}$$

The ratio of total pressure at the exit to that at the entrance can be computed from Eqs. (2–4) as

$$\frac{(P_t)_2}{(P_t)_1} = \left[\left(1 + \frac{\gamma - 1}{2} f\eta_0 M_s^2\right)\left(1 + \frac{\gamma - 1}{2} M_s^2\right)\right]^{\frac{\gamma}{\gamma-1}} \times \left[\frac{(\gamma + 1)M_1^2}{(\gamma - 1)M_1^2 + 2}\right]^{\frac{\gamma}{\gamma-1}}\left[\frac{\gamma + 1}{2\gamma M_1^2 - (\gamma - 1)}\right]^{\frac{1}{\gamma-1}} \tag{5}$$

where η_0 is given by Eq. (4) and the correction factor f is determined from experimental data as described in the following discussion.

III. Calculation of the Correction Factor f

The expression for $(P_t)_2/(P_t)_1$ given by Eq. (5) is a function of several parameters, i.e.,

$$(P_t)_2/(P_t)_1 = g(M_1,\eta_0,f) \tag{6}$$

It is possible to use this expression and experimental values for $(P_t)_2/(P_t)_1$ to find a simple empirical relationship between f and M_1. To this end, g is expanded in a Taylor's series about a nominal value of f. Retaining only the linear term results in

$$g(M_1,\eta_0,f) = g(M_1,\eta_0,f_n) + (f - f_n)\, \partial g/\partial f_{f=f_n} \tag{7}$$

where

$$\partial g/\partial f_{f=f_n} = \frac{\gamma\eta_0 M_s^2 g(M_1,\eta_0,f_n)}{2\left[1 + \left(\dfrac{\gamma - 1}{2}\right)\eta_0 f_n M_s^2\right]} \tag{8}$$

Substituting Eq. (8) into Eq. (7) and solving for $\epsilon \equiv f_n - f$, results in

$$\epsilon = \frac{2\left[1 + \left(\dfrac{\gamma - 1}{2}\right)\eta_0 f_n M_s^2\right]}{\gamma\eta_0 \cdot M_s^2}\left[1 - \frac{g(M_1,\eta_0,f)}{g(M_1,\eta_0,f_n)}\right] \tag{9}$$

The values used for $g(M_1,\eta_0,f)$ in Eq. (9) are the experimental results reported in Refs. 3–7. The remaining parameters are computed from the appropriate expressions given in the previous section. An initial value for f_n can be taken as unity, and the results iterated upon until convergence is achieved.

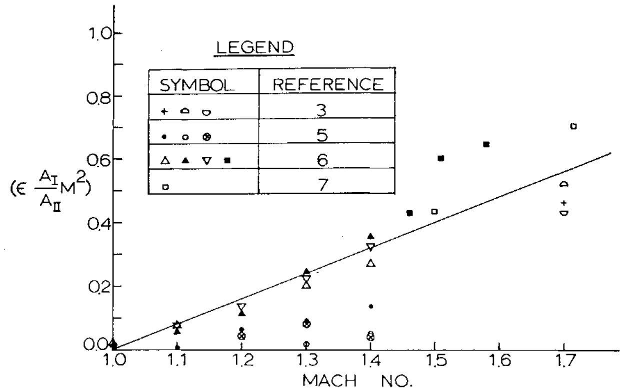

Fig. 2 Straight line approximation in the correction factor determination.

The resulting value for f is then

$$f = f_n + 2 \frac{\left[1 + \left(\frac{\gamma - 1}{2}\right) \eta_0 f_n M_s^2\right]}{\gamma \eta_0 M_s^2} \times \left[\frac{g(M_1,\eta_0,f)}{g(M_1,\eta_0,f_n)} - 1\right] \quad (10)$$

The results of Eq. (9) for various experimental data are plotted vs initial Mach number in Fig. 2. The plot of the product $\epsilon(A_I/A_{II})M_1^2$ vs Mach number is presented since it can be readily approximated by a straight line. The data used here represents a wide range of cascade geometries and test conditions as described in Table 1. The error represented by the scatter in the data about the straight line represents no more than $\pm 10\%$ in the value of the total pressure ratio $(P_t)_2/(P_t)_1$. The value of the correction factor based on the straight line fit of the data with f_n equal unity is

$$f = 1 + \tfrac{4}{5}(A_{II}/A_I)[(1/M_1^2) - 1/M_1] \quad (11)$$

Substitution of the results of Eqs. (3) and (11) for η_0 and f, respectively, into Eq. (5) results in the desired semiempirical relationship from which the total pressure ratio $(P_t)_2/(P_t)_1$ can be estimated.

IV. Concluding Remarks

A semiempirical expression for the prediction of supersonic compressor efficiency has been developed. The result is based on an assumed shock model and total pressure data taken from experimental tests on cascades of varying geometrical properties. The fact that the data can be reasonably represented by a straight line as a function of inlet Mach number makes the resulting efficiency expression attractive for design purposes.

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Optimal Stochastic Control and Aircraft Gust Alleviation

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Nomenclature

a_{cz}	= linear acceleration of aircraft center of mass in z direction, ft/sec²
C_{L_0}	= lift coefficient at trim angle of attack
g	= local acceleration due to gravity, ft/sec²
h	= aircraft altitude, ft
$K_1 \rightarrow K_7$	= control system gains
L_w	= gust characteristic length, ft
n	= $-a_{cz}/g$, normal acceleration factor
$\hat{q}$	= aircraft pitch rate
U_0	= aircraft forward velocity, fps
w_g	= gust vertical velocity, fps
xyz	= aircraft stability axis system (Ref. 2)
α_g	= $-w_g/U_0$, angle of attack perturbation due to gust, measured at aircraft center of mass, rad
η	= elevator angle, measured from trim, rad
Ω	= spatial frequency, rad/ft
σ_w	= rms vertical gust velocity, fps

Introduction

THE description of random atmospheric turbulence using semiempirical spectral models has facilitated the problem of calculating aircraft responses for flight through turbulent air. The design of automatic flight control systems to minimize mean square aircraft accelerations and to suppress certain elastic structural modes in level flight has been investigated. Little work has been done, however, in relating the optimal control system gains to the spectral characteristics of the gust field.

The research summarized in this Note has been directed toward the problem of minimizing the mean square normal acceleration of the center of mass of a large, rigid, jet transport flying level through a one-dimensional, turbulent upwash field.[1] In particular, the dependence of the mean square performance and optimal control system gains upon the gust

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